

# Mutually Exclusive Rules in LogicWeb

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LogicWeb has traditionally lacked devices for expressing mutually exclusive clauses. We address this limitation by adopting choice-conjunctive clauses of the form  $D_0 \& D_1$  where  $D_0, D_1$  are Horn clauses and  $\&$  is a linear logic connective. Solving a goal  $G$  using  $D_0 \& D_1 - pv(D_0 \& D_1, G)$  – has the following operational semantics: choose a successful one between  $pv(D_0, G)$  and  $pv(D_1, G)$ . In other words, if  $D_0$  is chosen in the course of solving  $G$ , then  $D_1$  will be discarded and vice versa. Hence, the class of choice-conjunctive clauses precisely captures the notion of mutually exclusive clauses.

## 1 Introduction

Internet computing is an important modern programming paradigm. One successful attempt towards this direction is LogicWeb[1]. LogicWeb is a model of the World Wide Web, where Web pages are represented as logic programs, and hypertext links represents logical implications between these programs. LogicWeb is an integral part of Semantic Web[2]. Despite much attractiveness, LogicWeb (and its relatives such as agent programming) has traditionally lacked elegant devices for structuring mutually exclusive rules. Lacking such devices, structuring mutually exclusive rules in LogicWeb relies on awkward devices such as the cut or *if-then-else* construct[9].

This paper proposes LinWeb, an extension to LogicWeb with a novel feature called choice-conjunctive clauses. This logic extends Horn clauses by the choice construct of the form  $D_0 \& D_1$  where  $D_0, D_1$  are Horn clauses and  $\&$  is a choice-conjunctive connective of linear logic. Inspired by [3], this has the following intended semantics: *choose* a successful one between  $D_0$  and  $D_1$  in the course of solving a goal. This expression thus supports the idea of mutual exclusion.

An illustration of this aspect is provided by the following clauses  $c1, c2$  which define the usual *max* relation:

$c1 : \max(X, Y, X) : - X \geq Y.$

$c2 : \max(X, Y, Y) : - X < Y.$

These two clauses are mutually exclusive. Hence, only one of these two clauses can succeed. Therefore, a more economical definition which consists of one clause  $c3$  is possible:

$c3 : (\max(X, Y, X) : - X \geq Y) \&$   
 $(\max(X, Y, Y) : - X < Y).$

This definition is more economical (and more deterministic) in the sense that it reduces the search space by cutting out the other alternatives. For example, consider a goal  $\max(9, 3, Max)$ . Solving this goal has the effect of choosing the first conjunct of (a copy of)  $c3$ , producing the result  $Max = 9$ . Our machine, unlike Prolog and other linear logic languages such as Lolli [6], does not create a

backtracking point for the second conjunct. The key difference between our language and other logic languages is that the selection action is present in our semantics, while it is not present at all in other languages.

The remainder of this paper is structured as follows. We describe LinWeb in the next section. In Section 3, we present some examples of LinWeb. Section 4 concludes the paper.

## 2 The Language

The language is an extended version of Horn clauses with choice-conjunctive clauses. It is described by  $G$ - and  $D$ -formulas given by the syntax rules below:

$$G ::= A \mid G \wedge G \mid D \supset G \mid \exists x G$$

$$D ::= A \mid G \supset D \mid \forall x D \mid D \& D$$

In the rules above,  $A$  represents an atomic formula. A  $D$ -formula is called a Horn clause with choice-conjunctive clauses.

In the transition system to be considered,  $G$ -formulas will function as queries and a set of  $D$ -formulas will constitute a program.

We will present an operational semantics for this language. The rules of LinWeb are formalized by means of what it means to execute a goal task  $G$  from a program  $\mathcal{P}$ . These rules in fact depend on the top-level constructor in the expression, a property known as uniform provability[7, 8]. Below the notation  $D; \mathcal{P}$  denotes  $\{D\} \cup \mathcal{P}$  but with the  $D$  formula being distinguished (marked for backchaining). Note that execution alternates between two phases: the goal-reduction phase (one without a distinguished clause) and the backchaining phase (one with a distinguished clause).

**Definition 1.** Let  $G$  be a goal and let  $\mathcal{P}$  be a program. Then the notion of executing  $\langle \mathcal{P}, G \rangle$  –  $pv(\mathcal{P}, G)$  – is defined as follows:

- (1)  $pv(A; \mathcal{P}, A)$ . % This is a success.
- (2)  $pv((G_1 \supset D); \mathcal{P}, A)$  if  $pv(\mathcal{P}, G_1)$  and  $pv(D; \mathcal{P}, A)$ .
- (3)  $pv(\forall x D; \mathcal{P}, A)$  if  $pv([t/x]D; \mathcal{P}, A)$ .
- (4)  $pv(D_0 \& D_1; \mathcal{P}, A)$  if choose a successful disjunct between  $pv(D_0; \mathcal{P}, A)$  and  $pv(D_1; \mathcal{P}, A)$ .
- (5)  $pv(\mathcal{P}, A)$  if  $D \in \mathcal{P}$  and  $pv(D; \mathcal{P}, A)$ . % change to backchaining phase.
- (6)  $pv(\mathcal{P}, G_1 \wedge G_2)$  if  $pv(\mathcal{P}, G_1)$  and  $pv(\mathcal{P}, G_2)$ .
- (7)  $pv(\mathcal{P}, \exists x G_1)$  if  $pv(\mathcal{P}, [t/x]G_1)$ .
- (8)  $pv(\mathcal{P}, D \supset G_1)$  if  $pv(\{D\} \cup \mathcal{P}, G_1)$

In the rule (4), the symbol  $D_0 \& D_1$  allows for the mutually exclusive execution of clauses. This rule can be implemented as follows: first attempts to solve the goal using  $D_0$ . If it succeeds, then do nothing (and do not leave any choice point for  $D_1$ ). If it fails, then  $D_1$  is attempted.

The following theorem connects our language to linear logic. Its proof is easily obtained from the discussions in [6].

**Theorem 1** *Let  $\mathcal{P}$  be a program and let  $G$  be a goal. Then,  $pv(\mathcal{P}, G)$  terminates with a success if and only if  $G$  follows from  $\mathcal{P}$  in intuitionistic linear logic.*

### 3 LinWeb

In our context, a web page corresponds simply to a set of  $D$ -formulas with a URL. The module construct *mod* allows a URL to be associated to a set of  $D$ -formulas. An example of the use of this construct is provided by the following “lists” module which contains some basic list-handling rules.

```
mod(www.dau.com/lists).
% deterministic version of the member predicate
memb(X, [X|L]) &
memb(X, [Y|L]) :- (neq X Y) ^ memb(X, L).
% optimized version of the append predicate
append([], L, L) &
append([X|L1], L2, [X|L3]) :- append(L1, L2, L3).
% the union of two lists without duplicates
uni([], L, L) &
uni([X|L], M, N) :- memb(X, M) ^ uni(L, M, N) &
uni([X|L], M, [X|N]) :- uni(L, M, N).
```

Our language makes it possible to change *memb* to be deterministic and more efficient: only one occurrence can be found. Our approach can be beneficial to most Prolog deterministic definitions. For example, the above definition of *append* explicitly tells the machine not to create a backtracking point. This is in contrast to the usual one in Prolog in which mutual exclusion must be inferred by the Prolog interpreter.

These pages can be made available in specific contexts by explicitly mentioning the URL via a hyperlink. For example, consider a goal  $www.dau.com/lists \supset uni([a, b], [b, c], Z)$ . This goal is translated to  $D_1 \supset D_2 \supset \dots uni([a, b], [b, c], Z)$  where each  $D_i$  is a  $D$ -formula in the *lists*. Solving this goal has the effect of adding each rule in *lists* to the program before evaluating  $uni([a, b], [b, c], Z)$ , producing the result  $Z = [a, b, c]$ .

### 4 Conclusion

In this paper, we have considered an extension to Prolog with mutually exclusive clauses. This extension allows clauses of the form  $D_0 \& D_1$  where  $D_0, D_1$  are Horn clauses. These clauses are particularly useful for replacing the cut in Prolog, making Prolog more efficient and more readable. We are investigating the connection between LinWeb and Japaridze’s computability logic [3, 4].

### 5 Acknowledgements

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